wjec cbac

GCE A LEVEL

1300U30-1

S23-1300U30-1

THURSDAY, 8 JUNE 2023 – MORNING

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

1300U301
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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Do not use pencil or gel pen. Do not use correction fluid.

Answer **all** questions.

Write your answers in the separate answer booklet provided, following the instructions on the front of the answer booklet.

Use both sides of the paper. Please only write within the white areas of the booklet.

Write the question number in the two boxes in the left hand margin at the start of each answer, e.g. | 0 | 1 |. Write the sub parts, e.g. **a**, **b** and **c**, within the white areas of the booklet.

Leave at least two line spaces between each answer.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Additional Formulae for 2023

Laws of Logarithms

$$\log_a x + \log_a y \equiv \log_a (xy)$$
$$\log_a x - \log_a y \equiv \log_a \left(\frac{x}{y}\right)$$
$$k \log_a x \equiv \log_a \left(x^k\right)$$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y \, \mathrm{d}x$

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

0 1 The 12th term of an arithmetic series is 41 and the sum of the first 16 terms is 488. Find the first term and the common difference of the series. [5]

0 2 **a)** Differentiate each of the following with respect to x.

i) $\left(\sin x + x^2\right)^5$ [2]

ii) $x^3 \cos x$ [2]

iii) $\dfrac{e^{3x}}{\sin 2x}$ [3]

b) Find the equation of the tangent to the curve

$$4y^2 - 7xy + x^2 = 12$$

at the point (2, 4). [6]

0 3 **a)** Express $\dfrac{9}{(1-x)(1+2x)^2}$ in terms of partial fractions. [4]

b) **Using your answer from part (a)**, find the expansion of $\dfrac{9}{(1-x)(1+2x)^2}$ in ascending powers of x as far as the term in x^2. State the values of x for which the expansion is valid. [7]

TURN OVER

0 4 A function f with domain $(-\infty, \infty)$ is defined by $f(x) = 6x^3 + 35x^2 - 7x - 6$.

a) Determine the number of roots of the equation $f(x) = 0$ in the interval [–1, 1]. [2]

b) Use the Newton-Raphson method to find a root of the equation $f(x) = 0$.
Starting with $x_0 = 1$,

i) write down the value of x_1,

ii) determine the value of the root correct to one decimal place. [4]

c) It is suggested that another iterative sequence

$$x_{n+1} = \sqrt{\frac{7x_n + 6 - 6x_n^3}{35}},$$

starting with $x_0 = -3$, could be used to find a root of the equation $f(x) = 0$.
Explain why this method fails. [2]

0 5 A tree is 80 cm in height when it is planted. In the first year, the tree grows in height by 32 cm. In each subsequent year, the tree grows in height by 90% of the growth of the previous year.

a) Find the height of the tree 10 years after it was planted. [4]

b) Determine the maximum height of the tree. [2]

0 6 **a)** Using the trigonometric identity $\cos(A+B)=\cos A\cos B-\sin A\sin B$, show that the **exact** value of $\cos 75°$ is $\dfrac{\sqrt{6}-\sqrt{2}}{4}$. [3]

b) Solve the equation $2\cot^2 x + \operatorname{cosec} x = 4$ for values of x between 0° and 360°. [6]

c) **i)** Express $7\cos\theta - 24\sin\theta$ in the form $R\cos(\theta+\alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

ii) Find all values of θ in the range $0° < \theta < 360°$ satisfying

$$7\cos\theta - 24\sin\theta = 5.$$

[6]

0 7 **a)** The graphs of $y = 5x-3$ and $y = 2x+3$ intersect at the point *A*. Show that the coordinates of *A* are (2, 7). [2]

b) On the same set of axes, sketch the graphs of $y = |5x-3|$ and $y = |2x+3|$, clearly indicating the coordinates of the points of intersection of the two graphs and the points where the graphs touch the x-axis. [4]

c) Calculate the area of the region satisfying the inequalities

$$y \geqslant |5x-3| \quad \text{and} \quad y \leqslant |2x+3|.$$

[4]

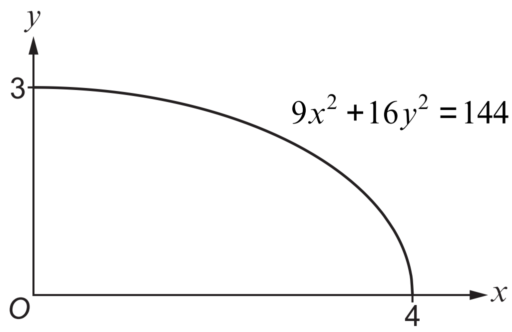
0 8 The function f is defined by $f(x) = \dfrac{4x^2+12x+9}{2x^2+x-3}$, where $x > 1$.

a) Show that $f(x)$ can be written as $2 + \dfrac{5}{x-1}$. [3]

b) Hence find the exact value of $\displaystyle\int_3^7 f(x)\,\mathrm{d}x$. [4]

TURN OVER

0 9 The aerial view of a patio under construction is shown below.



The curved edge of the patio is described by the equation $9x^2 + 16y^2 = 144$, where x and y are measured in metres.

To construct the patio, the area enclosed by the curve and the coordinate axes is to be covered with a layer of concrete of depth 0·06 m.

a) Show that the volume of concrete required for the construction of the patio is given by $0{\cdot}015\int_0^4 \sqrt{144 - 9x^2}\,\mathrm{d}x$. [3]

b) Use the trapezium rule with six ordinates to estimate the volume of concrete required. [4]

c) State whether your answer in part (b) is an overestimate or an underestimate of the volume required. Give a reason for your answer. [1]

1 0 Two real functions are defined as

$$f(x) = \frac{8}{x-4} \quad \text{for} \quad (-\infty < x < 4) \cup (4 < x < \infty),$$

$$g(x) = (x-2)^2 \quad \text{for} \quad -\infty < x < \infty.$$

a) **i)** Find an expression for $fg(x)$. [2]

ii) Determine the values of x for which $fg(x)$ does not exist. [3]

b) Find an expression for $f^{-1}(x)$. [3]

11 A curve C has equation $f(x) = 5x^3 + 2x^2 - 3x$.

a) Find the x-coordinate of the point of inflection. State, with a reason, whether the point of inflection is stationary or non-stationary. [5]

b) Determine the range of values of x for which C is concave. [2]

12 The rate of change of a variable y with respect to x is directly proportional to y.

a) Write down a differential equation satisfied by y. [1]

b) When $x = 1$ and $y = 0·5$, the rate of change of y with respect to x is 2. Find y when $x = 3$. [6]

13 The curve C_1 has parametric equations $x = 3p + 1$, $y = 9p^2$.

The curve C_2 has parametric equations $x = 4q$, $y = 2q$.

Find the Cartesian coordinates of the points of intersection of C_1 and C_2. [7]

14 **a)** Use integration by parts to evaluate $\int_0^1 (3x-1)e^{2x}\,dx$. [4]

b) Use the substitution $u = 1 - 2\cos x$ to find $\int \frac{\sin x}{1-2\cos x}\,dx$. [4]

END OF PAPER

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